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The Use of k Values in the Interpretation of Glass Density and Refractive Index Data

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ABSTRACT: A k value represents a type of summary statistic by means of which glass evidence may be interpreted. This statistic is defined as $k = V/D$, where the dispersion $V = (n_D - 1)/(n_F - n_C)$, and where D is the density. The k value is therefore the slope of a line through the 0,0 origin and a plot of dispersion versus density. Obtaining a k value facilitates the interpretation of commonality of source, since "within-item" variation in density and refractive index does not significantly alter the k value. "Between-item" variation, on the other hand, profoundly influences the k value, even if the density and one of the refractive indices are coincidentally similar.

KEYWORDS: forensic science, glass, density (mass/volume), refractive index

In the forensic science laboratory, glass evidence is routinely compared on the basis of density and refractive index. Many laboratories, but apparently not all, determine the dispersion characteristics of the evidence by determining the refractive index at the C line (486 nm) and the F line (656 nm) as well as the classically determined D line (589 nm).

The analytical techniques for the determination of density and refractive index values have been refined to the point where "within-item" variation exceeds, at least potentially, the precision and accuracy of the procedures used. For example, with respect to refractive index, the precision of the hot-stage technique is generally taken as $\pm 0.000\ 04$, whereas the variation between different locations in a single pane of untempered window glass is unlikely to exceed ± 0.0001 [1]. The presiding issue in the interpretation of whether two samples of glass could have shared a common source is therefore driven by "within-item" considerations.

Glass Evidence Criteria

Elmer Miller, formerly of the FBI laboratory, and probably the leading forensic glass expert of the 20th century, has, upon consideration of within-item variation, advocated certain criteria concerning the interpretation of glass evidence. He proposes [1] that to support the opinion that two samples are from a common source, the density must agree

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to within ± 0.001 g/cm³, and the refractive index must agree to within ± 0.0002 for the D line (589 nm) and ± 0.0004 for both the C (656 nm) and F lines (486 nm). In the discussion to follow, these values will be referred to as the "Miller Criteria." It is the position of the present author that these criteria are entirely sensible and appropriate, but are capable of further refinement.

A typical case is one in which the density and refractive index data are determined on one sample and then compared with corresponding data derived from another sample. But how is this comparison to proceed? Does the analyst "eyeball" the separate refractive index values, or is there benefit in comparing the dispersion, V , given by $V = (n_D - 1)/(n_F - n_C)$? And how *literally* should the Miller Criteria be applied? May the analyst accept agreement of $+0.0004$ for the F line refractive index in one instance but -0.0004 for another determination?

To attempt to answer these questions it is useful to examine the fundamental relationship between density and refractive index in solids. Electrostatic theory predicts that for compounds of generally similar composition, their densities and refractive indices will be highly correlated [2]. Glass samples of the same provenance will have very similar elemental composition and annealing histories, and individual samples from a common source may be legitimately viewed as representing members of a homologous series. In glass, as in other solid materials, many of the atoms will have unsatisfied bonds and will exist in an ionic form. For a homologous series of ions with randomly oriented point dipoles, density and refractive index are related [2] by

$$\frac{n^2 - 1}{4\pi + 4/3\pi(n^2 - 1)} = \sum_i N_i \alpha_i$$

where N_i is the number of ions per unit volume and α_i is the polarizability. (For purposes of the present discussion, it is only the factor N_i with which we need to be concerned. It may be assumed that the polarizability is independent of density; in solid materials such as glass, the overlapping of the charge distribution of vicinal ions will act to reduce the polarization interaction below that characteristic of point dipoles.)

This relationship between density and refractive index appears to have been overlooked by the forensic science community. The right-hand side of this equation is proportional to density through each ion N_i , but in this expression density is expressed as ions per unit volume. This relation can be made more explicit with respect to density by substituting in the equation the relationship $N_i = N_0 D f_i / M$, where N_0 is Avogadro's Number, D is density, f_i is the molar fraction of ions of type i , and M is the molecular weight. Density is now expressed in the more familiar form of grams per cubic centimetre (g/cm³).

The significance of this relationship for purposes of the present discussion is that if the density of a sample of glass were to be perceptibly increased as a result of a very slight change in its composition or annealing history, that is, the type of change associated with "within-item" variation, then an increase in its refractive index is *obligatory*. The converse is true as well; if by altering the composition or the thermal history of a sample we were to *decrease* its density, then a *decrease* in refractive index would be noted as well. Stated differently, one would not observe "within-item" variations in which a higher refractive index would be accompanied by a lower density or a lower refractive index accompanied by a higher density. (Those types of variations would, on the other hand, be consistent with "between-item" variation, indicating glass from different sources.)

Discussion

The Miller Criteria establish the bounds of variation that can be permitted between two samples from a common source. If we were to enter these criteria on a plot of density

versus refractive index (using the C or F line index as an example), it would appear as in Fig. 1. Any sample having values within the rectangle bounded by the Miller Criteria would ordinarily be interpreted as consistent with having shared a common origin. It is this idea that is being elaborated upon in the present work.

If we take electrostatic theory into account, then we can be much more discerning in what we will permit as satisfying "within-item" variation, and an avenue is opened up which may be exploited in the interpretation of glass evidence. Consider now a plot of density versus refractive index, still bounded by the Miller Criteria, but now honoring the requirements that any increase in refractive index must be accompanied by an increase in density and any decrease in refractive index must be accompanied by a decrease in density. The data points will no longer be scattered, but *will fall along a line* as depicted (for two samples of glass of different origins) in Fig. 2.

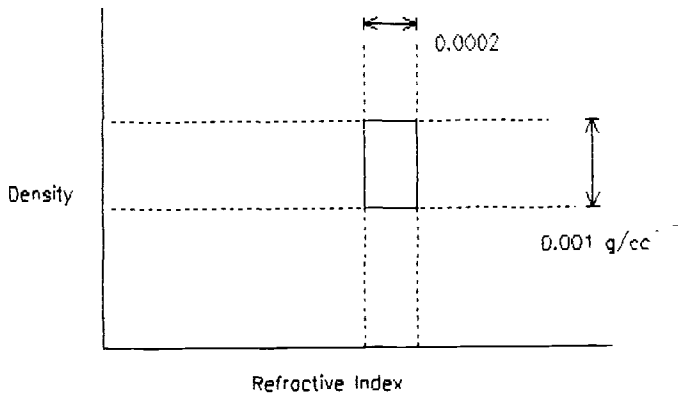


FIG. 1—Graphical representation of tolerance to "within-item" variation in glass density and refractive index. Any value within the rectangular "window" would be permissible if electrostatic theory is ignored. The dimensions of the window are given by the Miller Criteria.

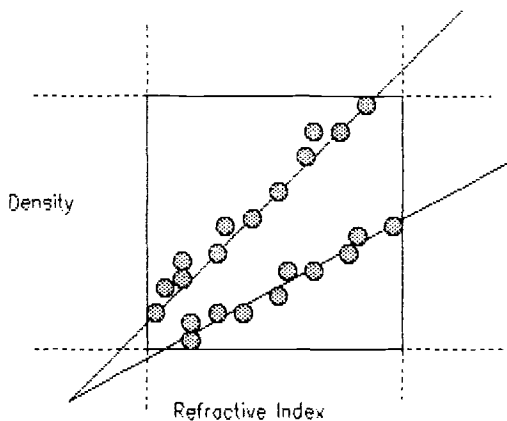


FIG. 2—Graphical representation of "within-item" variation if electrostatic theory is honored. Only those values falling along the k line would be interpreted as consistent with "within-item" variation. Values which are displaced from the line, even though they honor the conventional Miller Criteria, would be more consistent with "between-item" variation. In this illustration, permissible "within-item" variation is shown for two different types of glass, each with its own characteristic k slope.

Although within a given sample the precise values for density and refractive index may vary, the *slope* will not. The slope may therefore be considered a property of all samples of glass of the same provenance, that is, manufactured at the same time, with the same composition, and having received the same annealing treatment. This slope could be termed the k^0 value of the glass. (We will reserve the notation k for a different relationship as discussed below, that is, the slope of the line when density is plotted against dispersion.) Slight fluctuations in density and refractive index consistent with "within-item" variation should not significantly affect the k^0 value. On the other hand, differences in composition between two samples of glass arising from different provenance should markedly affect the k^0 value, *even if the density or the refractive index were coincidentally the same*.

One cannot, however, determine a k^0 value by running a line between a data point and the origin of any arbitrarily selected intersection of the axes of density and refractive index. Most plots seen of glass density and refractive index understandably ignore density below 2.47 and refractive index values below 1.513. To determine a valid k^0 value as defined here, the line must run through the 0,0 origin.

Additionally, in the view of the present author, it would not be a prudent professional practice to base any interpretation of glass evidence on a k^0 value using density and a single refractive index value; to do so would ignore the potentially useful contribution of dispersion.

The imperative necessity for collecting dispersion data on evidence samples is suggested by the data found in the FBI computer printout² of glass optical and physical properties. This data set lists well over 1000 samples of evidence glass, giving the density, dispersion, and the respective refractive indices for the C, D, and F lines.

Upon inspection of these data, it is apparent that there are numerous instances in which the density and the refractive index at the D line are shared by samples of glass that obviously could not have shared a common origin. Including refractive index data determined at the F and C lines helps to discriminate between samples, but even then there are samples (that clearly represent different sources) in the data set that cannot be distinguished. On this basis alone, the present writer concludes that it is not a prudent professional practice to compare glass samples on the basis of density and one refractive index, determined at the D line. To ignore the additional discrimination achieved if dispersion data is added to the data matrix is to invite interpretational error.

One can, however, determine a value, termed a k value, by plotting density, D , on the abscissa, versus dispersion, V , on the ordinate. (It should be recognized, however, that *plotting* the data is used here as a conceptual device to develop the rationale for determining the k value. In practice, the k value is determined computationally rather than graphically, by simply dividing the dispersion value, V , by the density, D . The quotient is the tangent of the slope of the k line.)

The k value so derived may now be used in the interpretation of glass evidence in the manner described below. A few examples gleaned from the FBI data will serve as illustrations; these are depicted in Table 1.

Consider the glass that appears in the FBI data set as Glass 222 (Entry 1 in Table 1). The k value (that is, V 60.0233 divided by D 2.4870) for this glass is 24.1348. Consider now a hypothetical sample from the same source, in which the density and all three refractive indices were elevated by the maximum amount permitted by the Miller Criteria. Although the values are now different, this new glass would still be considered under the Miller Criteria as the "same" glass as the original. The k value for this sample (Entry 2 in Table 1) is now 24.1345, a shift of only 0.0003. Suppose now that the density and refractive indices were depressed by the maximum amount permitted by the Miller Criteria. This sample (Entry 3 in Table 1) would now have a k value of 24.1352, a shift of

²E. Miller, personal communication, FBI Laboratory, Washington, DC. 1979.

TABLE 1—Effect on the *k* value of altering glass density and refractive index values by the maximum amount permitted by the Miller Criteria. Entry 1 is FBI Glass 222; all other entries are hypothetical samples, but in no instance does the manipulation of values from the parent Sample 222 exceed the Miller Criteria for “within-item” variation. Stated differently, all of the entries in this table represent samples that would fall within the permissible rectangular window depicted in Fig. 1.

Entry	Density	<i>n_F</i>	<i>n_D</i>	<i>n_C</i>	<i>V</i>	<i>k</i>	Δ <i>k</i>
1 ^a	2.4870	1.5223	1.5162	1.5137	60.0233	24.1348	...
2 ^b	2.4880	1.5227	1.5164	1.5141	60.0465	24.1345	0.0003
3 ^c	2.4860	1.5219	1.5160	1.5133	60.0000	24.1352	0.0004
4 ^d	2.4870	1.5227	1.5162	1.5133	54.9149	22.0808	2.0540
5 ^e	2.4880	1.5227	1.5162	1.5133	51.9149	22.0719	2.0629
6 ^f	2.4760	1.5223	1.5162	1.5141	62.9269	25.4147	1.2799
7 ^g	2.4880	1.5219	1.5160	1.5133	60.0000	24.1158	0.0190
8 ^h	2.4860	1.5227	1.5164	1.5141	60.0465	24.1539	0.0191

^aParent sample, FBI Glass 222.
^bAll values elevated by the amount permitted by the Miller Criteria.
^cAll values depressed by the amount permitted by the Miller Criteria.
^dDensity and *n_D* unchanged, *n_F* elevated, *n_C* depressed.
^eDensity and *n_F* elevated, *n_D* unchanged, *n_C* depressed.
^fDensity depressed, *n_F* and *n_D* unchanged, *n_C* elevated.
^gDensity elevated, all other values depressed.
^hDensity depressed, all other values elevated.

0.0004. These two samples are still varying only to the extent permitted by “within-item” variation, but are honoring the rules that electrostatic theory imposes. If we honor those rules in our conceptualization of this subject, namely, *density and refractive index vary only in sympathy with one another*, then we can refine the Miller Criteria still further for purposes of interpretation.

Consider now several hypothetical samples of glass with *dissimilar* origins. These new samples may coincidentally agree with one or more of the values of FBI Glass 222, but because of differences in composition or annealing history have different dispersion curves or a different level of correlation between density and any one of the refractive indices. These glass samples will have their own *k* line, but as sample of glass from different sources they need not follow the *k* line of the first sample.

The first dissimilar glass is one in which the density and the D line refractive index is identical to FBI Glass 222, but the C line refractive index is depressed and the F line index is elevated (Entry 4 in Table 1); the depression or elevation is the maximum amount permitted by the Miller Criteria, but does not exceed the Miller Criteria. The *k* value for this dissimilar glass is now 22.0808, a shift of 2.0540 from the original. It should be stressed that the variation here does not at any time exceed the Miller Criteria for “within-item” variation; the dissimilar glass, however, does have a steeper dispersion curve, and this is reflected in a profound difference in the *k* value. Instead of a shift of 3 or 4 in the fourth decimal place, as for “same source” samples, we now have a shift of 20 540 in the fourth decimal place of the *k* value. Two other hypothetical glass samples of “different source” origin are depicted in Entries 5 and 6 of Table 1.

Consider now two additional hypothetical samples of dissimilar origin. We may elevate the density and depress all three refractive indices (Entry 7 of Table 1), or depress the density and raise the refractive indices (Entry 8 in Table 1). The shift in *k* value (relative to the parent FBI Glass 222 sample) in these two instances is approximately 0.0190. These two samples, in which the relative relationship of the refractive indices at the C, D, and F lines are maintained, represent particularly severe tests of the *k* value concept.

TABLE 2—Two samples from the FBI data set that could not be discriminated by the conventional Miller Criteria, but which can be easily discriminated by their k values. The difference in k values between the two samples is 0.9581.

Density	n_F	n_D	n_C	V	k
2.4895	1.5231	1.5178	1.5152	65.5443	26.3283
2.4890	1.5235	1.5178	1.5153	63.1463	25.3701

since the dispersion tends to remain poised. Still, the shift in k value is on the order of 190 in the fourth decimal place, as opposed to the shift of 3 or 4 in the fourth decimal place seen in Entries 2 and 3.

In reviewing the FBI data, the greatest shift in k value resulting from altering the density and refractive index values by the Miller Criteria (while at the same honoring electrostatic theory) is encountered in FBI Glass 20. With this sample, elevating all of the values by the maximum amount permitted by the Miller Criteria results in a shift of 0.0009 in k value. It is significant that the FBI Glass 20 has the highest dispersion, that is, the lowest V value, of the entire FBI list.

Conclusion

On the basis of the foregoing, the present author proposes that any shift in k value of 0.001 or less be construed as consistent with "within-item" variation, but that a shift greater than 0.001 in k value is due to "between-item" variation and more consistent with glass of a different provenance.

One final example from the FBI data is depicted in Table 2. The first glass listed is from a 1977 California case, and the second is from a 1975 Connecticut case—obviously different glasses. The values for density and all three refractive indices are quite similar, and if one were to use the Miller Criteria without allowing for the expression of electrostatic theory, these samples would be interpreted as consistent with having shared a common origin. Using the k value concept, however, the k values are 26.3282 and 25.3701, respectively, a difference in 9581 in the fourth decimal place. This is greatly in excess of the 10 in the fourth decimal place that is proposed as the cutoff between "within-item" and "between-item" variation, and the two samples can therefore be easily discriminated by the k values.

References

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